Honors Pre-Calculus and Pre-Calculus Summer Packet

Terri Price

I am excited about the new school year and having you as a student in Honors or Regular Pre-Calculus class. We will be working hard this year as you learn many new concepts- I hope we can have a little fun along the way!

To prepare yourself for Pre-calculus, you should complete this summer packet. All of the concepts in this packet have been covered in previous math classes and provide an overview of the essential concepts necessary for success in Honors Pre-Calculus or Pre-Calculus. This packet is not designed to be busy work, but designed to help make the transition to the next level an easier one by giving you the opportunity to refresh material that you may have forgotten from previous math classes.

Students are expected to print out their own copy of this packet, show all their work in the packet, and circle their answers. REMEMBER NO WORK, NO CREDIT!! The packets will be collected and graded the first full day of school and given a homework grade. Throughout the first week of school we will review the concepts covered in the packet and a quiz will be given on selected topics from this packet.

I have included notes and examples throughout the packet to help refresh your knowledge of the material. If more examples are needed on any topic presented, I suggest a search on the Internet for the specific topic. There is an abundance of helpful and Free math resources that can be found online (a great resource is Khan Academy). I will hold a help sessions for the summer packet Wednesday, July 31, 2019 from 10:00 am to 12:00 pm in the media center classroom at Trinity Presbyterian School. Students attending this session should have attempted the problems prior to the session and use the help session to ask about specific questions and concepts.

In addition to the summer packet I would encourage all 11th grade Honors Pre-Calculus and Pre-Calculus students to log into their Khan Academy account, link their account with their College Board Account, and work on the personalized PSAT/SAT practice for the 2019 Fall PSAT.

I hope each of you have a WONDERFUL summer and return to school in August prepared to conquer the Mathematics World of Pre-Calculus!!!!

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Honor's Precalculus and Precalculus

Radicals:
To simplify means that 1) no radicand has a perfect square factor and
2) there is no radical in the denominator (rationalize).

Recall – the **Product Property** $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24} = \sqrt{4} \cdot \sqrt{6}$ find a perfect square factor
$= 2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ split apart, then multiply both the numerator and the denominator by $\sqrt{2}$

$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$ multiply straight across and simplify

If the denominator contains 2 terms –
multiply the numerator and the denominator by the ***conjugate*** of the denominator
The ***conjugate*** of $3 + \sqrt{2}$ is $3 - \sqrt{2}$ (the sign changes between the terms).

Simplify each of the following.

1. $\sqrt{32}$
2. $\sqrt{(2x)^8}$
3. $\sqrt[3]{-64}$
4. $\sqrt{49m^2n^8}$
5. $\sqrt{\frac{11}{9}}$
6. $\left(\sqrt{5} - \sqrt{6}\right)\left(\sqrt{5} + \sqrt{2}\right)$

Rationalize.

7. $\frac{1}{\sqrt{2}}$
8. $\frac{3}{2 - \sqrt{5}}$

Complex Numbers:

Form of complex number - $a+bi$

Where a is the "real" is part and bi is the "imaginary" part

Always make these substitutions $\sqrt{-1}=i$ and $i^2=-1$

- To simplify: pull out the $\sqrt{-1}$ before performing any operation

Example: $\sqrt{-5}=\sqrt{-1}\cdot\sqrt{5}$ Pull out $\sqrt{-1}$

$= i\sqrt{5}$ Make substitution

Example: $(i\sqrt{5})^2 = i\sqrt{5}\cdot i\sqrt{5}$ List twice

$= i^2\sqrt{25}$ Simplify

$=(-1)(5)=-5$ Substitute

- Treat i like any other variable when +, -, ×, or ÷ (but always simplify $i^2=-1$)

Example: $2i(3+i)=2(3i)+2i(i)$ Distribute

$= 6i+2i^2$ Simplify

$= 6i+2(-1)$ Make substitution

$= -2+6i$ Simplify and rewrite in complex form

- Since $i=\sqrt{-1}$, no answer can have an 'i' in the denominator **RATIONALIZE!!**

Simplify.

9. $\sqrt{-49}$

10. $6\sqrt{-12}$

11. $-6(2-8i)+3(5+7i)$

12. $(3-4i)^2$

13. $(6-4i)(6+4i)$

Rationalize.

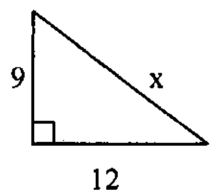
14. $\dfrac{1+6i}{5i}$

Geometry:

Pythagorean Theorem (right triangles): $a^2 + b^2 = c^2$

Find the value of x.

15.



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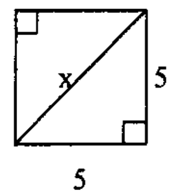

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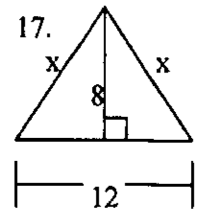
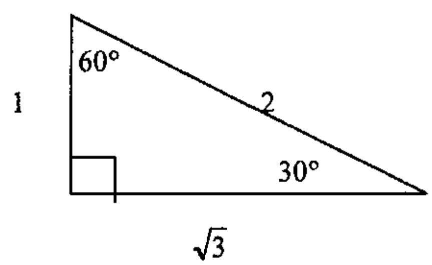
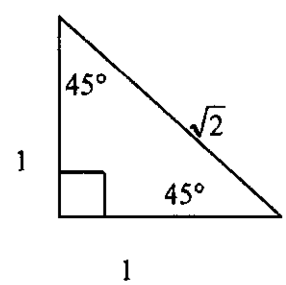


18. A square has perimeter 12 cm. Find the length of the diagonal.

* In $30° - 60° - 90°$ triangles, sides are in proportion $1, \sqrt{3}, 2$.

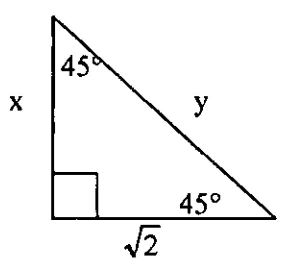


*In $45° - 45° - 90°$ triangles, sides are in proportion $1, 1, \sqrt{2}$.



Solve for x and y.

19.



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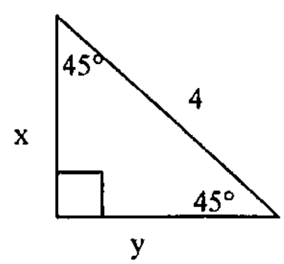

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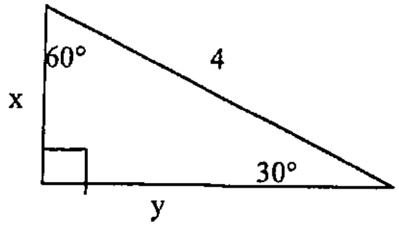

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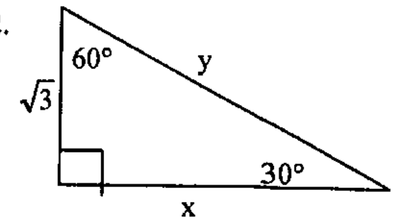


Equations of Lines:

Slope intercept form: $y = mx + b$	**Vertical line:** x = c (slope is undefined)
Point-slope form: $y - y_1 = m(x - x_1)$	**Horizontal line:** y = c (slope is 0)
Standard Form: $Ax + By = C$	**Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1}$

23. State the slope and y-intercept of the linear equation: $5x - 4y = 8$.

24. Find the x-intercept and y-intercept of the equation: $2x - y = 5$

25. Write the equation in standard form: $y = 7x - 5$

Write the equation of the line in slope-intercept form with the following conditions:

26. slope = -5 and passes through the point (-3, -8)

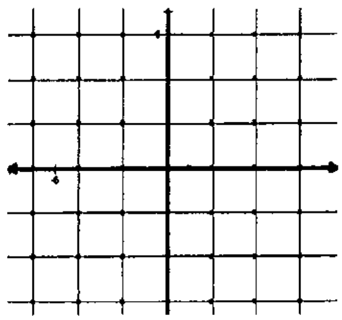
27. passes through the points (4, 3) and (7, -2)

28. x-intercept = 3 and y-intercept = 2

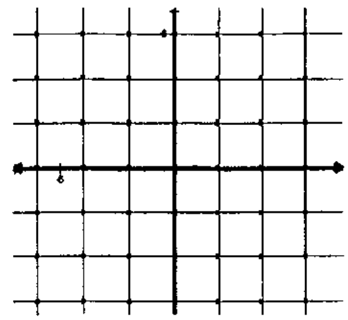
Graphing:

Graph each function, inequality, and / or system.

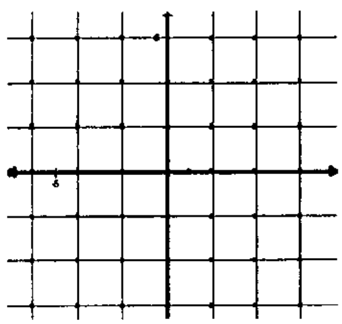
29. $3x - 4y = 12$



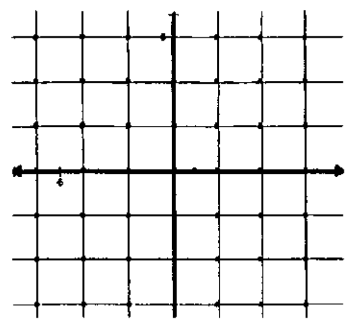
30. $\begin{cases} 2x + y = 4 \\ x - y = 2 \end{cases}$



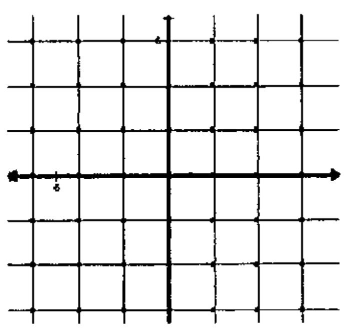
31. $y < -4x - 2$



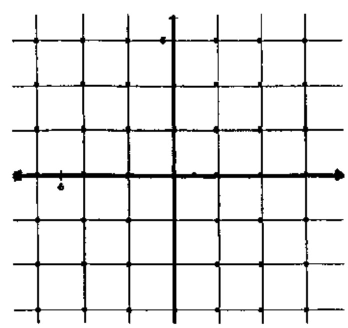
32. $y + 2 = |x + 1|$



33. $y > |x| - 1$



34. $y + 4 = (x - 1)^2$



Vertex: ________________

x-intercept(s): ________________

y-intercept(s): ________________

Systems of Equations:

$3x + y = 6$
$2x - 2y = 4$

Substitution:
Solve 1 equation for 1 variable.
Rearrange.
Plug into 2nd equation.
Solve for the other variable.

Elimination:
Find opposite coefficients for 1 variable.
Multiply equation(s) by constant(s).
Add equations together (lose 1 variable).
Solve for variable.

Then plug answer back into an original equation to solve for the 2nd variable.

$y = 6 - 3x$	solve 1st equation for y
$2x - 2(6 - 3x) = 4$	plug into 2nd equation
$2x - 12 + 6x = 4$	distribute
$8x = 16$	simplify
$x = 2$	

$6x + 2y = 12$	multiply 1st equation by 2
$\underline{2x - 2y = 4}$	coefficients of y are opposite
$8x = 16$	add
$x = 2$	simplify

Plug $x = 2$ back into original

$3(2) + y = 6$
$6 + y = 6$
$y = 0$

Solve each system of equations. Use any method.

35. $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

36. $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

37. $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$

Exponents:

TWO RULES OF ONE
1. $a^1 = a$

Any number raised to the power of one equals itself.

2. $1^a = 1$

One to any power is one.

ZERO RULE
3. $a^0 = 1$

Any nonzero number raised to the power of zero is one.

PRODUCT RULE
4. $a^m \cdot a^n = a^{m+n}$

When multiplying two powers that have the same base, add the exponents.

QUOTIENT RULE
5. $\dfrac{a^m}{a^n} = a^{m-n}$

When dividing two powers with the same base, subtract the exponents.

POWER RULE
6. $(a^m)^n = a^{m \cdot n}$

When a power is raised to another power, multiply the exponents.

NEGATIVE EXPONENTS
7. $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$

Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

38. $5a^0$

39. $\dfrac{3c}{c^{-1}}$

40. $\dfrac{2ef^{-1}}{e^{-1}}$

41. $\dfrac{(n^3p^{-1})^2}{(np)^{-2}}$

Simplify.

42. $3m^2 \bullet 2m$

43. $(a^3)^2$

44. $(-b^3c^4)^5$

45. $4m(3a^2m)$

Polynomials:

To add / subtract polynomials, combine like terms.

EX: $8x-3y+6-(6y+4x-9)$ *Distribute the negative through the parantheses.*

$=8x-3y+6-6y-4x+9$ *Combine terms with similar variables.*

$=8x-4x-3y-6y+6+9$

$=4x-9y+15$

Simplify.

46. $3x^3+9+7x^2-x^3$

47. $7m-6-(2m+5)$

To multiplying two binomials, use FOIL.

EX: $(3x-2)(x+4)$ *Multiply the first, outer, inner, then last terms.*

$=3x^2+12x-2x-8$ *Combine like terms.*

$=3x^2+10x-8$

Multiply.

48. $(3a + 1)(a - 2)$

49. $(s + 3)(s - 3)$

50. $(c - 5)^2$

51. $(5x + 7y)(5x - 7y)$

Factoring.

Follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

a.) If you have one (other than 1) factor it out front.
b.) If you don't have one, move on to STEP 2.

STEP 2: How many terms does the polynomial have?

2 Terms

a.) Is it difference of two squares? $a^2 - b^2 = (a+b)(a-b)$

EX: $x^2 - 25 = (x+5)(x-5)$

b.) Is it sum or difference of two cubes? $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

EX: $m^3 + 64 = (m+4)(m^2 - 4m + 16)$
$p^3 - 125 = (p-5)(p^2 + 5p + 25)$

3 Terms

$x^2 + bx + c = (x+\quad)(x+\quad)$	**Ex:** $x^2 + 7x + 12 = (x+3)(x+4)$
$x^2 - bx + c = (x-\quad)(x-\quad)$	$x^2 - 5x + 4 = (x-1)(x-4)$
$x^2 + bx - c = (x-\quad)(x+\quad)$	$x^2 + 6x - 16 = (x-2)(x+8)$
$x^2 - bx - c = (x-\quad)(x+\quad)$	$x^2 - 2x - 24 = (x-6)(x+4)$

4 Terms -- Factor by Grouping

a.) Pair up first two terms and last two terms
b.) Factor out GCF of each pair of numbers.
c.) Factor out front the parentheses that the terms have in common.
d.) Put leftover terms in parentheses.

Ex: $$\begin{aligned} x^3 + 3x^2 + 9x + 27 &= \left(x^3 + 3x^2\right) + \left(9x + 27\right) \\ &= x^2\left(x+3\right) + 9\left(x+3\right) \\ &= \left(x+3\right)\left(x^2+9\right) \end{aligned}$$

Factor completely.

52. $z^2 + 4z - 12$

53. $6 - 5x - x^2$

54. $2k^2 + 2k - 60$

55. $-10b^4 - 15b^2$

56. $9c^2 + 30c + 25$

57. $9n^2 - 4$

58. $27z^3 - 8$

59. $2mn - 2mt + 2sn - 2st$

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use quadratic formula.

EX: $x^2 - 4x = 21$ — *Set equal to zero FIRST.*

$x^2 - 4x - 21 = 0$ — *Now factor.*

$(x+3)(x-7) = 0$ — *Set each factor equal to zero.*

$x + 3 = 0 \quad x - 7 = 0$ — *Solve each for x.*

$x = -3 \quad x = 7$

Solve each equation.

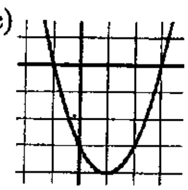
60. $x^2 - 4x - 12 = 0$

61. $x^2 + 25 = 10x$

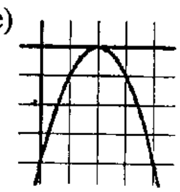
62. $x^2 - 14x + 40 = 0$

DISCRIMINANT: The number under the radical in the quadratic formula $(b^2 - 4ac)$ can tell you what kinds of roots you will have.

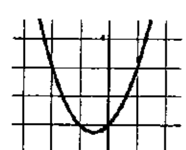
IF $b^2 - 4ac > 0$ you will have TWO real roots.
(touches x-axis twice)



IF $b^2 - 4ac = 0$ you will have ONE real root
(touches the x-axis once)



IF $b^2 - 4ac < 0$ you will have TWO imaginary roots.
(Graph does not cross the x-axis)



QUADRATIC FORMULA – **allows you to solve any quadratic for all its real and imaginary roots.** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EX: Solve the equation: $x^2 + 2x + 3 = 0$

$$Solve: \quad x = \frac{-2 \pm \sqrt{-8}}{2}$$
$$x = \frac{-2 \pm 2i\sqrt{2}}{2}$$
$$x = -1 \pm i\sqrt{2}$$

Solve each quadratic.
Use EXACT values.

63. $x^2 - 9x + 14 = 0$

64. $5x^2 - 2x + 4 = 0$

Roots = ______________________

Roots = ______________________

Long Division – can be used when dividing any polynomials.
Synthetic Division – can ONLY be used when dividing a polynomial by a linear (degree one) polynomial.

EX: $\dfrac{2x^3+3x^2-6x+10}{x+3}$

Long Division

$\dfrac{2x^3+3x^2-6x+10}{x+3}$

$$
\begin{array}{r}
2x^2-3x+3+\dfrac{1}{x+3} \\
=x+3\overline{\smash{)}\;2x^3+3x^2-6x+10} \\
\underline{(-)\;(2x^3+6x^2)\quad\quad\quad} \\
-3x^2-6x\quad\; \\
\underline{(-)\;(-3x^2-9x)\quad} \\
3x+10 \\
\underline{(-)\;(3x+9)} \\
1
\end{array}
$$

Synthetic Division

$\dfrac{2x^3+3x^2-6x+10}{x+3}$

$$
\begin{array}{r|rrrr}
-3 & 2 & 3 & -6 & 10 \\
 & \downarrow & -6 & 9 & -9 \\
\hline
 & 2 & -3 & 3 & 1
\end{array}
$$

$= 2x-3x+3+\dfrac{1}{x+3}$

Divide each polynomial using long division OR synthetic division.

65. $\dfrac{c^3-3c^2+18c-16}{c^2+3c-2}$

66. $\dfrac{x^4-2x^2-x+2}{x+2}$

To evaluate a function for a given value, simply plug the value into the function for x.

Evaluate each function for the given value.

67. $f(x)=x^2-6x+2$

$f(3)=$ ______

68. $g(x)=6x-7$

$g(x+h)=$ ______

69. $f(x)=3x^2-4$

$5\left[f(x+2)\right]=$ ______

Composition and Inverses of Functions:

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "*f* of *g* of *x*" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x)=2x^2+1$ *and* $g(x)=x-4$ find *f(g(x)).*

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2+1 \\ &= 2(x^2-8x+16)+1 \\ &= 2x^2-16x+32+1 \\ f(g(x)) &= 2x^2-16x+33 \end{aligned}$$

Suppose $f(x)=2x,\ g(x)=3x-2,$ *and* $h(x)=x^2-4$. **Find the following:**

70. $f[g(2)]=$ _______

71. $f[g(x)]=$ _______

72. $f[h(3)]=$ _______

73. $g[f(x)]=$ _______

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse, $f^{-1}(x)$, if possible.

74. $f(x)=5x+2$

75. $f(x)=\frac{1}{2}x-\frac{1}{3}$